Optimal Electricity Trading Strategy for a Household Microgrid

Zhaoming Qin, Haochen Hua, Hong Liang, Randa Herzellah, Yuyang Zhou, and Junwei Cao

Abstract— **The recent integration of distributed generators (DGs) and renewable energy sources (RESs) into the power system led to the manifestation of a significant number of household microgrid (MG) systems in the electricity market. However, in most of the cases, the DGs in the MG system are passive and are not equipped by their own controllers, thus their integration increases the fluctuation in the power system and brings challenges to its management and control. To address these challenges, this paper proposes a novel electricity trading strategy for a household MG. This will be achieved by formulating a nonlinear stochastic control problem that will then be solved such that the profit through electricity trading is maximised. To solve this optimisation problem, a gradient descent method based on compressive sensing is applied. Finally, some numerical examples are given to illustrate the effectiveness of the proposed control method. The results from the simulation experiments indicate that the proposed electricity trading strategy achieves the target and satisfies all constraints by controlling the energy router (ER) with the energy storage component.**

NOMENCLATURE

MG	Microgrid.
RES	Renewable energy source.
DG	Distributed generator.
EI	Energy Internet.
ER	Energy router.
SDE	Stochastic differential equation.
ODE	Ordinary differential equation.
CIR	Cox-Ingersoll-Ross.
CS-GDM	Compressive sensing-baced gradient descent method
P_R	RESs output power.
P_L	Load output power.
P_{ER}	power of ER's energy storage component.

I. INTRODUCTION

With the rapid development of renewable energy resourses (RES), the RES based distributed generators (DG) have recently been widely exploited in the power systems, especially in the household microgrid (MG) systems [1]. For instance, in Decemebr 2019 there have been more than 1025,000 solar photovoltaic panels installed in the UK, which provided about 13356MW capacity in total. Furthermore, more than half of the power were generated from the small-sized types equipment, which are less than 5MW [2]. It can be predicted that in the future, with the increasing application of the small and medium-sized MG, there will be a significant revolution in the power system field. This revolution will help the chance of evolving more renewable and clean energy to avoid the environmental pollution. On the other hand this revolution in the power sytem structure will bring many challenges that will need to be dealt with [3].

In order to better cope with the upcoming revolution in the power system field and face the unpredictable challenges, the new concept of the energy internet (EI) has been defined [4]. According to the EI concept, the future electricity market is characterized as open and equal [5]. As a crucial part of the future electricity market, household MG has drawn much attention [6]-[8]. Typically, the household MG consists of Photovoltaic panels, smart electricity meter, household battery (energy storage equipment), household loads, and other smart equipment [9]. Thus, MG can be considered as both the energy provider and the energy consumer. This kind of dual-role means that the rights can be switched between these two roles in the middle of the trade of the future electricity market. The proposed formulation will allow the market to optimize the distribution of benefits by itself and then realize a more efficient and fair distribution of benefits [10]. Therefore, MG has been one of the major interesting research subjects and considerable literature have extensively investigated it. Paper [11] provided an intelligent scheduling strategy based on the mixed-integer nonlinear programming to optimize the operation schedule of building energy systems. This proposed optimal scheduling strategy minimizes the overall operation cost by considering the uncertainty of the operation energy cost and the cost concerning the plant on/off penalty. The possibility of smoothing out the load variance in a household MG by regulating the charging patterns of family PHEVs is investigated in [12]. In order to facilitate domestic DR to effectively respond to pressures in energy prices and distribution network conditions, [13] presented a novel dispatch strategy for shared ownership of domestic energy storage batteries between customers and DNOs.

Another key feature of the future electricity market is the fluctuation of electricity prices. Compared with the fixed prices in the traditional power system, the electricity prices in the future electricity market will change dynamically over time along with the production cost and generation capacity. Besides, the future EI can provide the service accordingly based on the analysis of the costumer's electricity bill data to maximize the value of the electricity [15]. Regarding the electricity price forecasting, a hybrid electricity price prediction methodology by combining WT with ARMA,

Z. Qin, H. Hua, H. Liang and J. Cao are with Beijing National Research Center for Information Science and Technology, Tsinghua University, Beijing, 100084, P. R. China,

R. Herzellah and Y. Zhou are with the Systems Analytics Research Institute, Aston University, Aston Triangle, Birmingham B4 7ET, UK. e-mail: annamada@163.com, r.herzallah@aston.ac.uk

Corresponding Author: Junwei Cao jcao@tsinghua.edu.cn

KELM, and SAPSO is provided in [16].

As the mediator that MG communicates with outside and exchanges energy, energy router (ER) plays a vital role in the EI [17]. Paper [18] designed an AC-DC hybrid multi-port energy router for power distribution networks, which consists of high voltage AC port, low voltage AC port and low voltage DC port while [21] offered formal verification solutions for an ER-based system by proposing a continuous-time Markov chain model characterizing the architecture of the ER-based system.

Motivated by the information mentioned above, in this paper, we propose an electricity trading strategy for household MG system without having to have controllable DGs. Considering the uncertainty of the power dynamics of RESs, and loads and electricity prices stochastic differential or difference equations provide the most complete description under these conditions [14], [19], [20]. Therefore in this work SDEs are applied to model their deviation. It is notable that, to capture mean reversion and sharp spikes observed in electricity price [22], we model the electricity price as a nonlinear SDE called the CIR diffusion process. ODEs are utilized to model the power dynamics of ER. Then, the considered MG system is written as a nonlinear SDE. To maximize the profit through electricity transactions, a nonlinear stochastic optimization problem is formulated with constraints for ER and controller. Because of the complexity of nonlinear stochastic problem, a gradient descent method based on compressive sensing called CS-GDM method is adopted. In order to show the feasibility of the proposed approach, numerical examples are conducted. The primary importance and contribution of this paper are highlighted as follows.

- This is the first time that the electricity trading strategy is investigated by characterizing the electricity price as a nonlinear SDE. The CIR diffusion process can describe the electricity price more precisely.
- By adopting the compressive sensing based approach, we are able to deal with the complex nonlinear stochastic problem. Most of the existing methods, e.g., the Monte Carlo method, failed to solve this problem effectively.
- Compared to the existing fixed electricity model, the proposed control scheme is more practical for the future electricity market.

The rest of this paper is organized as follows. Section II describes the model of a typical household. Section III formulates the MG operation and electricity trading problem into a nonlinear stochastic optimization problem. In Section IV, a gradient descent method based on compressive sensing is adopted to solve the problem. Section V presents the numerical results. Finally, Section VI concludes the paper.

II. SYSTEM MODELLING

In this paper, a typical household MG is investigated. As shown in Fig. 1, the MG normally consists of RESs, loads and an ER. In such a MG, all the power generation relies on RESs. The ER in the considered MG plays two roles: on the one hand, the MG can sell energy to the utility company or purchase energy from the utility company through ER; on the other hand, the ER can function as a battery, which means the ER can store energy and discharge.

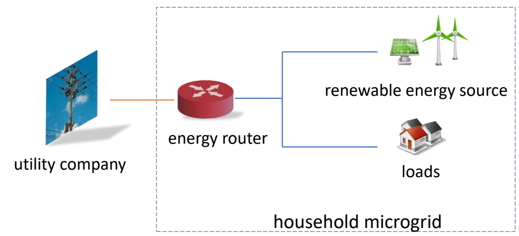

Fig. 1. A typical household microgird.

For notation simplicity, unless necessary, time t of all variables throughout this paper is omitted.

To characterise the uncertainty and fluctuation of power of RESs and loads, the linear stochastic differential equations is adopted as follows [23]

$$dP_R = -\theta_r(P_R - \mu_r)dt + \sigma_r dW_r, \tag{1}$$
$$dP_L = -\theta_l(P_L - \mu_l)dt + \sigma_l dW_l, \tag{2}$$

where W_r and W_l represent scalar Wiener processes, parameters μ_r and μ_l refer to expected value of P_R and P_L, parameters θ_r, σ_r, θ_l and σ_l are weights of the deterministic and stochastic terms of dP_R and dP_L, respectively. All the system parameters can be determined by parameter estimation methods [24].

Similar with [25], we apply the following linear ordinary differential equations to describe the power dynamics of ER's storage component in the considered MG

$$dP_{ER} = -\frac{1}{T_{ER}}(P_{ER} - B_{ER}u)dt, \tag{3}$$
$$dS_{ER} = \eta P_{ER} dt. \tag{4}$$

In (3), T_{ER} is the time constant of ER's energy storage component, B_{ER} represents the maximum absolute value of P_{ER}, and u is the control input signal. Besides, S_{ER} in (4) denotes the state of charge of ER's energy storage component, while η in (4) is a coefficient related to charge/discharge efficiency. In addition, the constraints for u and S_{ER} are shown as follows,

$$-1 \le u \le 1, \tag{5}$$
$$C_{min} \le S_{ER} \le C_{max}. \tag{6}$$

where C_{min} and C_{max} are the allowed minimum and maximum value of S_{ER}, respectively.

Due to the non-elasticity of demand and renewable electricity generation, electricity prices exhibit large spikes. Moreover, the electricity price is influenced by generation cost, power quality and other factors. Therefore, in the future electricity market, the electricity price is similar to the price of stocks, rather than a constant value [26]. To describe the fluctuation of the electricity price, in this paper, a nonlinear SDE is applied. Motivated by Cox–Ingersoll–Ross (CIR) diffusion process [27], electricity price between the utility company and the considered MG is formulated as follows,

$$d\lambda = -\theta_p(\lambda - \mu_p)dt + \sigma_p\sqrt{\lambda}dW_p, \tag{7}$$

where λ is the electricity price, μ_p stands for the expected value of λ. Parameters θ_p and σ_p are the weights of the deterministic and stochastic terms respectively of $d\lambda$. The square root term in the stochastic differential equation 7 emphasises that the dynamics of the price are in fact nonlinear.

Denote $x = [P_R, P_L, \lambda, P_{ER}, S_{ER}]^T$, $W = [W_r, W_l, W_p]^T$, the system can be rewritten in the following form,

$$dx = (Ax + Bu + C)\,dt + D(x)dW, \tag{8}$$

where,

$$A = \begin{bmatrix} -\theta_r & 0 & 0 & 0 & 0 \\ 0 & -\theta_l & 0 & 0 & 0 \\ 0 & 0 & -\theta_p & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{T_{ER}} & 0 \\ 0 & 0 & 0 & \eta & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{B_{ER}}{T_{ER}} \\ 0 \end{bmatrix}.$$

$$C = \begin{bmatrix} \theta_r\mu_r \\ \theta_l\mu_l \\ \theta_p\mu_p \\ 0 \\ 0 \end{bmatrix}, D = \begin{bmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_l & 0 \\ 0 & 0 & \sigma_p\sqrt{\lambda} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

In this sense, our investigated MG dynamics have been transformed into a stochastic control system (8). It is noted that matrix $D(x)$ is a time variant parameter matrix related to state $x(t)$. The term $D(x)dW$ indicates that system (8) is indeed a nonlinear system.

III. PROBLEM FORMULATION

In this section, the energy trading strategy for the considered MG is formulated as an optimal control problem. The details will be given as follows.

To maintain power balance of investigated MG, the following equation can be obtained

$$P_R = P_L + P_{ER} + P_{out}, \tag{9}$$

where P_{out} denotes power exchange between the MG and utility company. There are three situations regarding to P_{out}. 1. when P_{out} is greater than zero, the MG sells energy to utility company; 2. when P_{out} is zero, the MG maintains power balance without power exchange; 3. when P_{out} is smaller than zero, the MG purchases energy from utility company.

As an energy trading strategy, our target is to maximize the profit through energy trading. With the above mentioned notation, $\lambda(t)P_{out}(t)$ represents the profit for considered MG per unit time at time t. Thus, considering a time period T, the objective function, whose value should be minimized, is defined as follows,

$$\mathcal{J}_0 = \mathbb{E}\int_0^T -\lambda P_{out}dt, \tag{10}$$

where $\mathbb{E}$ represents the mathematical expectation, and the integral denotes transactions from the MG to the utility company in time period T. By substituting equality (9) into objective function (10), the new form is presented as

$$\begin{aligned}\mathcal{J}_0 =& \mathbb{E}\int_0^T -\lambda(P_R - P_L - P_{ER})dt \\ &= \mathbb{E}\int_0^T x^TQxdt,\end{aligned} \tag{11}$$

where,

$$Q = \frac{1}{2}\begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The optimal control problem is then formulated as

$$\begin{aligned} \min_u \quad & \mathcal{J}_0 \\ \text{s.t.} \quad & (1) - (7) \end{aligned} \tag{12}$$

In order to deal with the inequality constraints (5) and (6), a penalty term $\mathcal{P}$ is introduced to the objective function as follows

$$\begin{aligned}\mathcal{P} =& \epsilon_1(S_{ER} - \frac{C_{min} + C_{max}}{2})^2 \\ &+ \epsilon_2\left(-\log(u+1) - \log(1-u)\right),\end{aligned} \tag{13}$$

where ϵ_1 and ϵ_2 are weight coefficients. In (13), the first line, the quadratic form, is related to constraint (6). The second line in (13) is a logarithmic form. For a constraint $f(x) \le 0$, the logarithmic barrier function $\phi(x) = -\log(-f(x))$ is differentiable and closed: it goes to ∞ as $f(x)$ approaches zero [28]. Note that the approximation to primal control problem (12) becomes more accurate as ϵ_2 decrease to zero.

In this manner, the optimal control problem (12) is transformed into an equality contrained problem (14) shown as follows,

$$\begin{aligned} \min_u \quad & \mathcal{J} = \mathbb{E}\int_0^T \left\{x^TQx + \mathcal{P}\right\}dt \\ \text{s.t.} \quad & (1) - (4), \quad (7) \end{aligned} \tag{14}$$

It is notable that the objective function is nonlinear because of the logarithmic barrier function.

By solving the stochastic nonlinear optimal control problem (14), the optimal energy trading strategy for the considered household MG can be obtained. The detailed methodologies will be presented in next section.

IV. SOLUTION TO THE CONTROL PROBLEM

In this section, we will introduce the numerical method that was proposed in our previous work to solve the stochastic optimization problem (14). The algorithm, named as CS-GDM, combines the traditional gradient descent method with compressive sensing. To use this method, we firstly need to discretize the Wiener process in (8). Here we use the Haar basis to construct the Wiener process [29].

The multilevel Haar functions $H_k^{(n)}$ are defined as $H_0^{(0)}(t) = 1$ and

$$H_k^{(n)}(t) = 2^{\frac{n-1}{2}}\psi(2^{n-1}(t) - k), \quad n \geq 1, \\ k \in \{0, 1, 2, \ldots, 2^{n-1} - 1\}, \quad for \quad t \in [0, 1]. \tag{15}$$

where n is the level, and ψ is the mother Haar function:

$$\psi(t) = \begin{cases} 1, & t \in [0, 1/2); \\ -1, & t \in [1/2, 1); \\ 0, & \text{otherwise.} \end{cases}$$

The Wiener process can then been approximated by

$$W = \sum_{n=1}^{\infty} \sum_{k} \omega_k^{(n)} \int_0^t H_k^{(n)}(s) ds. \tag{16}$$

Here $\omega_k^{(n)}$ are the random variables selected from the independent identical standard Gaussian distribution. For the numerical calculation, we consider the N-term truncation which satisfies the accuracy requirements of the problem.

After the discretization, the solution of the state equation (8) can be expanded with a set of stochastic polynomials [30], which is given by,

$$x(t) = \sum_{j=1}^{p} c_j(t)\Psi_j, \tag{17}$$

where $\{\Psi_j\}_{j=1}^p$ are the multi-Hermite polynomials, p is the number of polynomials, and $\{c_j(t)\}_{j=1}^p$ are the coefficients which need to be determined by the compressive sensing method. This means that we can recover the sparse coefficients in (17) by solving the following basis pursuit denoising (BPDN) problem as

$$\hat{c} = \text{arg min } \|c\|_1, \qquad \text{subject to} \quad \|X - \Psi c\| \leq \epsilon, \tag{18}$$

where X is the sample simulation results, Ψ is the information matrix formed by inserting the stochastic sample points into the Hermite polynomials, and $c = \{c_j\}_{j=1}^p$ is the coefficient vector to be determined.

Compressive sensing method provides an efficient algorithm for solving the stochastic state equation (8), which has always been considered as the main computation complexity when using iterative methods to solve the control problem (14).

To use the gradient descend method to solve the optimal control problem, we need to introduce the Hamiltonian function H [31] :

$$H = \mathcal{J} + \lambda'(t)\{Ax(t) + Bu + C + D(x)dW\}, \tag{19}$$

where $\lambda(t)$ is the system co-state. Then, the equivalent Hamiltonian systems for the optimal control problem are [32]:

$$dx = Ax + Bu + C + D(x)dW, \tag{20}$$

$$d\lambda = -\frac{\partial \mathcal{J}}{\partial x} - (A + \frac{\partial D}{\partial x})'\lambda \tag{21}$$

$$\frac{\partial H}{\partial u} = \frac{\partial \mathcal{J}}{\partial x} + B\lambda. \tag{22}$$

As the system (20)-(22) is nonlinear, we employ our CS-GMD algorithm to solve this stochastic optimization problem numerically. The detailed procedure of the algorithm can be summarized as:

Algorithm 1 ⟨CS-GDM optimization algorithm ⟩

The optimization algorithm is as follows:

a. Discretize the Wiener process with N terms in (16), then use multi-Hermite polynomials and expand the state variable $x(t)$.
b. Introduce the Hamiltonian function and deduce the equivalent systems (20)-(22) of the optimization problem (14).
c. Initialize the gradient descend parameters.
d. During the iteration, use compressive sensing to solve equation (20) to get the state variable $x(t)$.
e. From $x(t)$, use compressive sensing to solve equation (21) to get the co-state variable $\lambda(t)$.
f. Form $x(t)$ and $\lambda(t)$ derive the gradient $\frac{\partial H}{\partial u}$ in (22).
g. Calculate the objective function $\mathcal{J}$. if the relative error reach the requirement, then finish the iteration. If not, go back to step d.

The CS-GDM method we proposed can save computational cost dramatically compared with some other methods such as the Monte Carlo method. The reduction in the computational requirements is achieved because the proposed compressive sensing method can achieve the same accuracy with much fewer samples than other sampling methods. The relevant numerical results will be given in the next section.

V. NUMERICAL EXAMPLE

To demonstrate the proposed strategy, the considered household MG system is simulated in this section and the control performance of the proposed energy trading strategy is evaluated.

The simulation period is $t \in [0, 60]$ min, with 1 min time increment. The related optimal stochastic control problem is solved using MATLAB 2019a environment.

The parameters of the considered MG system (1)-(7) are shown in Table I.

Motivated by stochastic differential equation (1) and (2), simulation for fluctuation of P_R and P_L are depicted in Fig. 2 and Fig. 3, respectively. For the reason that the initial value of P_L is set to be 15 KW and the expected value μ_l is set to 10 KW, there is a downward trend in the curve of P_L in Fig. 3.

TABLE I
SYSTEM PARAMETERS FOR SIMULATION

Parameters	Value	Parameters	Value	Parameters	Value
θ_r	2	μ_r	15 KW	σ_r	2
θ_l	1.5	μ_l	10 KW	σ_l	1.5
θ_p	0.5	μ_p	0.15\$/KWh	σ_p	0.1
T_{ER}	1 h	B_{ER}	4 KW	η	0.2
ϵ_1	2	ϵ_2	1		

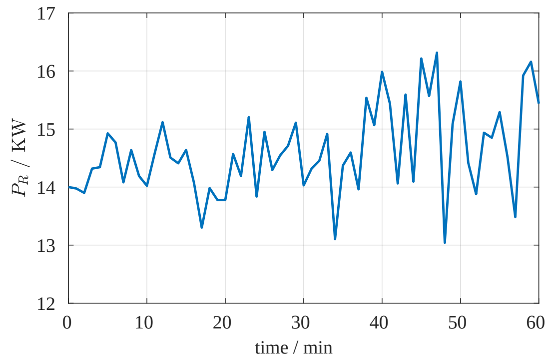

Fig. 2. Fluctuation curve of P_R within a hour.

Modelled by nonlinear stochastic differential equation (7), the electricity price λ is presented in Fig. 4. It can be clearly seen from Fig. 4 that there are more drastic spikes in the last 30 minutes.

With the proposed energy trading strategy, the power and state of charge of ER's energy storage component are plotted in Fig. 5. It is clear that the state of charge is restricted within a suitable range during the investigated time period. In addition, Fig. 5, shows that in the interval $[0, 15]$ min, the power P_{ER} is greater than zero, and the state of change S_{ER} increases. However, in the interval $[15, 60]$ min, the power P_{ER} is smaller than zero, and the state of change S_{ER} decreases. Fig .5 indicates that the ER absorbs energy in the first quarter and discharges in the remaining time.

The profit of the household MG through electricity trading is illustrated in Fig. 6. It is clear that during time $[0, 10]$ min,

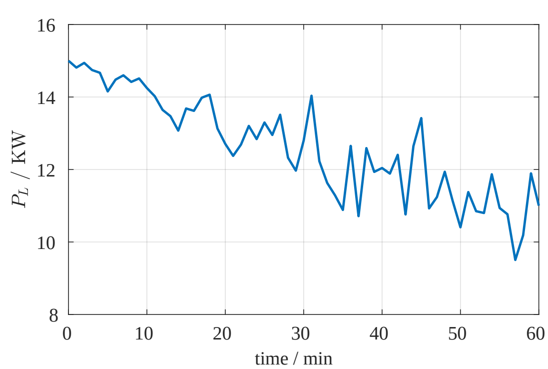

Fig. 3. Fluctuation curve of P_L within a hour.

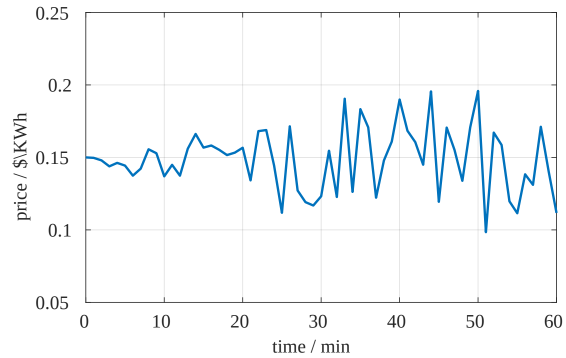

Fig. 4. Fluctuation curve of price λ within a hour.

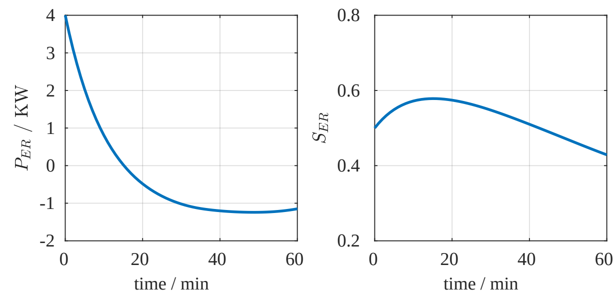

Fig. 5. Fluctuation curve of P_{ER} and S_{ER} within a hour.

the power of RESs is smaller than the power of loads, and then the household MG purchases energy from the utility company. After 10 min, P_R is greater than P_L, leading to selling energy from the MG.

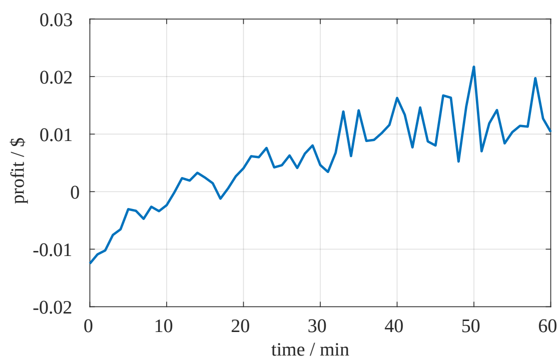

Fig. 6. Profit for household MG through electricity trading per minute.

The numerical examples provided above properly shows the feasibility of the proposed energy trading strategy, and the efficacy of such method is evaluated.

VI. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

In this paper, an optimal electricity trading strategy for a household MG system without controllable DGs is proposed.

By a compressive sensing based method, this stochastic optimization control problem is solved. In the simulation section, comparison indicates that the optimal electricity trading strategy has a better performance. The state of charge of ER's energy storage component is controlled within a suitable range, and the objective of maximizing the profit through electricity trading has been achieved.

VII. ACKNOWLEDGMENTS

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