Stochastic Robust H_∞ Control Strategy for Coordinated Frequency Regulation in Energy Internet Considering Time Delay and Uncertainty

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***Abstract*—In this article,we consider the coordinatedfrequency controlissues for energy Internet (EI) within a certain area. We assume multiple AC microgrids (MGs) are interconnected via energy routers (ERs) and electricity is transmitted between MGsvia DC transmission technology, in the sense that the AC bus frequency in each individual MG is independent. In addition, the scenario of EIis presumed to bedisconnected with the main grid. We formulate a generalized connection topology for the considered EI scenario.Each MG is assumed to be composed of a combination of distributed wind turbine generators (WTGs), photovoltaic (PV) units, micro turbines (MTs), fuel cells (FCs), battery energy storage (BES) devices and local loads. Then we formulate the dynamics of the above components asa class of ordinary differential equations (ODEs) and stochastic differential equations (SDEs) with parameter uncertainty and time-varying delay in the state.To alleviate frequency fluctuation in each individual MG, the coordinated frequency control problem is formulated as problems of robust stabilizationandrobust H_∞ control. The linear matrix inequality (LMI) approach isapplied to solve thesetwo problems sufficiently with desired state feedback controllers obtained. Numerical simulations are illustrated to show the effectiveness of the proposed method.**

Keywords-* H_∞ *control; energy Internet; frequency control;linear matrix inequality;stochastic systems; time-delay systems; uncertain systems

I. INTRODUCTION

In the past decades, microgrids (MGs) have been popular for power management in rural areas which are difficult to get accessto the main grid, due to expensive installation costs oftransmission lines [1]. The MG contains distributedrenewable energy resources (RERs) such as wind turbine generators (WTGs) and photovoltaic (PV) units, conventionalpower generationdevices such as diesel engine generators, micro turbines (MTs) and fuel cells (FCs), energy storage devices such as battery energy storage (BES) devices and flywheel energy storage devices, and local load devices. At present, for the existing MG projectsin the world, read can consult, e.g., [1]–[3], etc.

For a single islanded AC MG, due to the defects of power generation by RERs such as intermittent, nonlinear, stochastic, uncertain, etc., power balance between power generation and usage is difficult to be achieved [4], [5]. Such power deviation would cause AC bus frequency oscillation and the possible worst result is power blackout [6], [7].

To achieve a better power balance, theconcept of energy Internet (EI) is proposed accompany with development in renewable energyutilization and modern information and communication technology[8]–[10].Following the core router of network technology,energy routers (ERs) (also known as energy exchange devices or energy hubs) can be utilized to exchange energy between the interconnectedMGs in EI [11]–[13]. Within the scenario of EI, each individual MG shall be able to share the capacity of their energy storage devices with the other connected ones [10], [14].Recently, distributed economic modelpredictive control of EI for coordinated stochasticenergy management is investigated in [15]. For an EI disconnected with the main grid, coordinated dispatch in multiple cooperative autonomous islanded MGs has been studied in [16].

On the other hand, power system frequency control issues have received much attention [17]. The deterministicH_∞control technique can be applied to alleviate frequency deviation for an islanded MG [18]. In [19] the problems of frequency regulation and life-time extension for BES devices are investigated for an islanded AC MG and are solvedviadeterministic mixedH_2/H_∞control technique. Note that most of the previous literatures regarding control problems in EI, e.g., [18]–[20], etc., do not consider system parameter uncertainty, time delay and stochasticity, which objectively exist in power system [17].How to design a class of controllersfor the scenario of EI considering system parameter uncertainty, time delay, and stochastic power deviation,such that the frequency in each individual MG can be controlled coordinately remains an open problem.

In this paper, we consider a scenario of EI which is composed of WTGs, PV units, MTs, FCs, BES devices and loads. First, a typical topology for EI is illustrated and we emphasize such topology can be extended to a generalized version of EI. The dynamics of each component in the multiple MGs are modeled as differential equations. Since power generation by WTGs and PV units is randomly affected by wind power and solar irradiation, respectively, and load power deviations depend on residents' custom of power usage, we model the power dynamics of WTGs, PV units and loads as stochastic differential equations (SDEs). The dynamics of MTs, FCs, ERs, BES devices and AC bus frequency deviations are modeled as ordinary differential equations (ODEs). The desired controllers are designed in MTs, FCs and ERs.For real

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industrial scenarios, time delaycaused bycommunication and equipment response always exists when control signal is implementedto the power system[17].In addition, since parameter uncertainty is unavoidable in power system [17], especially when linearized models are applied to describe the power system dynamics, we reveal such estimation errors by introducing parameter uncertainty in the coefficients of EI system's dynamical equations.Then we rewrite the dynamics of each component in EI as one explicit state space equation. Next,aiming to regulate the frequencies in EI coordinately, we formulate two control problems: robust stabilization and robust H_∞control.Based on the control techniques introduced in [21], we solve these two problems via linear matrix inequality (LMI) approach with desired controllers obtained. Based on real engineering data,we give numerical simulations to demonstrate the effectiveness of the proposed method.

We emphasize that the importance of this work is that we are working on an application of modern robust H_∞control theory for uncertain stochastic systems with state delay with respect to the scenario of EI. In this paper, the real engineering problem is modeled and solved by mathematical techniques. The contribution of this paper can be outlined as follows:

1) A *generalized* topology for EI is provided with a special case studied in detail.
2) The coordinated frequency control problem in EI is considered in the premise that *all* the dynamics of the components in multiple MGs are considered.
3) This is the very first time thatsystem stochasticity, time delay and parameter uncertainty are *simultaneously* considered in the system modeling of the dynamics in EI.
4) The problem of coordinated frequency regulation in EI is appropriately*formulated* as problems of robust stabilization and robust H_∞control.

The rest of the paper is organized as follows: Section II introduces the EI system modeling. Problem formulation and solutions are introduced in Section III. In Section IV, numerical examples are illustrated. Finally, Section V concludes the paper.

Notation: The notation to be used in the paper is standard. The identity and null matrices are denoted by I and 0, respectively. $X > 0(\geq, < 0)$ denotes that X is a positive definite (positive semi-definite, negativedefinite) matrix. $X > Y$ means that $X - Y$ is positive definite.

II. SYSTEM MODELING

In this section, we establish a typicalmathematical model forEI system and use differential equations to express the dynamic performances of each EI component.

Consider the scenario of EI consisting of four AC MGs (MG_1, MG_2, MG_3 and MG_4) interconnected via four ERs, described in Fig.1. We assume that each MG include one local load and one BES device ($\mathrm{L}_1, \mathrm{L}_2, \mathrm{L}_3$and L_4represent for load devices in MG_1, MG_2, MG_3 and MG_4, respectively; $\mathrm{BES}_1, \mathrm{BES}_2, \mathrm{BES}_3$ and BES_4 represent for BES devices in MG_1,MG_2,MG_3 and MG_4, respectively). The loads and BES devices areconsidered uncontrollable and passively responding to the AC bus frequency deviation. Besides, we assume thatMG_1 comprises one PV unit (PV_1), one FC (FC_1) and one WTG (WTG_1); MG_2 comprises one MT (MT_2) and one PV unit (PV_2); MG_3 comprises one FC (FC_3); and MG_4 contains one MT (MT_4) and one WTG (WTG_4). We assume that electricity transmitted among MGsis converted to DC power in advance, thus the frequency in each single MG is independent. ERs are used to transmit DC power between the interconnected MGs and to response to the control signals.

In Fig. 1, we observe that MG_4 is connected withMG_3 only, MG_1 and MG_2 are connected with two other MGs, and MG_3 is connected with all of the remainingMGs. Such topology can be extended to a generalized EI structure containing almost every interconnecting modes. In general, given an EI composed of $n$interconnected MGs, then for one MG, any connection mode can be described as one MG connecting to one,or two, or up to $n-1$ numbers of other MGs. Based on such topology, we claim that our research is representative and can be applied in a variety of real engineering situations.

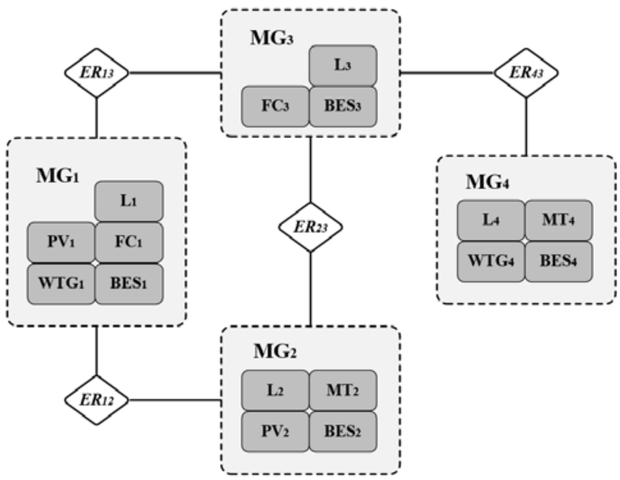

Fig. 1. The scenario of a typical EI.

Throughout this paper, we denote ΔP_{L_1}, ΔP_{PV_1},ΔP_{WTG_1},ΔP_{FC_1} andΔP_{BES_1} as the power changes ofL_1, PV_1, WTG_1, FC_1 and BES_1 in MG_1, respectively; we denoteΔP_{L_2}, ΔP_{PV_2}, ΔP_{MT_2} and ΔP_{BES_2} as the power changes of L_2, PV_2, MT_2 and BES_2 in MG_2, respectively;we denote ΔP_{L_3}, ΔP_{FC_3} and ΔP_{BES_3} as the power changes ofL_3, FC_3 and BES_3in MG_3, respectively;we denote ΔP_{L_4}, ΔP_{WTG_4}, ΔP_{MT_4} and ΔP_{BES_4} as the power changes of L_4, WTG_4, MT_4 and BES_4 in MG_4, respectively. Notations Δf_1, Δf_2,Δf_3 and Δf_4 stand for the AC bus frequency deviations in MG_1, MG_2, MG_3 and MG_4, respectively.Notations ER_{12}, ER_{13}, ER_{23} and ER_{43} stand for ERs, and the direction of power flows is expressed by the sequence of the routers' numbered corner marks. For example, ER_{12}implies that power is assumed to be transmitted from MG_1 toMG_2.

Time constants of all theMG components are denoted as T_{L_1}, T_{L_2},T_{L_3}, T_{L_4},T_{PV_1}, T_{PV_2}, T_{WTG_1}, T_{WTG_4}, T_{MT_2}, T_{MT_4}, T_{FC_1}, T_{FC_3}, T_{BES_1}, T_{BES_2}, T_{BES_3}, T_{BES_4}, $T_{ER_{12}}$, $T_{ER_{13}}$, $T_{ER_{23}}$

and$T_{ER_{43}}$ corresponding to theirsubscripts.To describe AC bus frequency coefficients,M_1, M_2,M_3andM_4 stand for the inertia constantsof MG_1 , MG_2 , MG_3 and MG_4 , respectively, whereasD_1, D_2,D_3andD_4 stand for the damping coefficients of MG_1, MG_2 , $\mathrm{MG}_3$and MG_4,respectively.

$$\begin{cases} \mathrm{d}\Delta P_{L_1} = \left[-\left(\frac{1}{T_{L_1}}+\Delta O_{L_1}\right)\Delta P_{L_1}+\frac{1}{T_{L_1}}v_{L_1}\right]dt \\ \qquad +r_{L_1}\Delta P_{L_1}\mathrm{d}W(t), \\ \mathrm{d}\Delta P_{PV_1} = \left[-\left(\frac{1}{T_{PV\,1}}+\Delta O_{PV_1}\right)\Delta P_{PV_1}+\frac{1}{T_{PV\,1}}v_{PV_1}\right]dt \\ \qquad +r_{PV_1}\Delta P_{PV_1}\mathrm{d}W(t), \\ \mathrm{d}\Delta P_{WTG_1} = -\left(\frac{1}{T_{WTG\,1}}+\Delta O_{WTG_1}\right)\Delta P_{WTG_1}dt \\ \qquad +\frac{1}{T_{WTG\,1}}v_{WTG_1}dt+r_{WTG_1}\Delta P_{WTG_1}\mathrm{d}W(t), \\ \Delta\dot{P}_{FC_1} = -\frac{1}{T_{FC\,1}}\Delta P_{FC_1}(t-\tau(t)) \\ \qquad +\frac{1}{T_{FC\,1}}(b_{FC_1}+\Delta b_{FC_1})u_{FC_1}, \\ \Delta\dot{P}_{BES\,1} = -\frac{1}{T_{BES\,1}}\Delta P_{BES_1} \\ \qquad +\frac{1}{T_{BES\,1}}(r_{BES_1}+\Delta r_{BES_1})\Delta f_1, \\ \Delta\dot{f}_1 = -\frac{2D_1}{M_1}\Delta f_1+\frac{2}{M_1}\Delta P_1. \end{cases} \tag{1}$$

$$\begin{cases} \mathrm{d}\Delta P_{L_2} = \left[-\left(\frac{1}{T_{L_2}}+\Delta O_{L_2}\right)\Delta P_{L_2}+\frac{1}{T_{L_2}}v_{L_2}\right]dt \\ \qquad +r_{L_2}\Delta P_{L_2}\mathrm{d}W(t), \\ \mathrm{d}\Delta P_{PV_2} = \left[-\left(\frac{1}{T_{PV\,2}}+\Delta O_{PV_2}\right)\Delta P_{PV_2}+\frac{1}{T_{PV\,2}}v_{PV_2}\right]dt \\ \qquad +r_{PV_2}\Delta P_{PV_2}\mathrm{d}W(t), \\ \Delta\dot{P}_{MT_2} = -\frac{1}{T_{MT\,2}}\Delta P_{MT_2}(t-\tau(t)) \\ \qquad +\frac{1}{T_{MT\,2}}(b_{MT_2}+\Delta b_{MT_2})u_{MT_2}, \\ \Delta\dot{P}_{BES\,2} = -\frac{1}{T_{BES\,2}}\Delta P_{BES_2} \\ \qquad +\frac{1}{T_{BES\,2}}(r_{BES_2}+\Delta r_{BES_2})\Delta f_2, \\ \Delta\dot{f}_2 = -\frac{2D_2}{M_2}\Delta f_2+\frac{2}{M_2}\Delta P_2. \end{cases} \tag{2}$$

$$\begin{cases} \mathrm{d}\Delta P_{L_3} = \left[-\left(\frac{1}{T_{L_3}}+\Delta O_{L_3}\right)\Delta P_{L_3}+\frac{1}{T_{L_3}}v_{L_3}\right]dt \\ \qquad +r_{L_3}\Delta P_{L_3}\mathrm{d}W(t), \\ \Delta\dot{P}_{FC_3} = -\frac{1}{T_{FC_3}}\Delta P_{FC_3}(t-\tau(t)) \\ \qquad +\frac{1}{T_{FC_3}}(b_{FC_3}+\Delta b_{FC_3})u_{FC_3}, \\ \Delta\dot{P}_{BES\,3} = -\frac{1}{T_{BES\,3}}\Delta P_{BES_3}+\frac{1}{T_{BES\,3}}(r_{BES_3}+\Delta r_{BES_3})\Delta f_3, \\ \Delta\dot{f}_3 = -\frac{2D_3}{M_3}\Delta f_3+\frac{2}{M_3}\Delta P_3. \end{cases} \tag{3}$$

$$\begin{cases} d\Delta P_{L_4} = \left[-\left(\frac{1}{T_{L_4}}+\Delta O_{L_4}\right)\Delta P_{L_4}+\frac{1}{T_{L_4}}v_{L_4}\right]dt \\ \qquad +r_{L_4}\Delta P_{L_4}dW(t), \\ \mathrm{d}\Delta P_{WTG_4} = -\left(\frac{1}{T_{WTG\,4}}+\Delta O_{WTG_4}\right)\Delta P_{WTG_4}dt \\ \qquad +\frac{1}{T_{WTG\,4}}v_{WTG_4}dt+r_{WTG_4}\Delta P_{WTG_4}\mathrm{d}W(t), \\ \Delta\dot{P}_{MT_4} = -\frac{1}{T_{MT\,4}}\Delta P_{MT_4}(t-\tau(t)) \\ \qquad +\frac{1}{T_{MT\,4}}(b_{MT_4}+\Delta b_{MT_4})u_{MT_4}, \\ \Delta\dot{P}_{BES\,4} = -\frac{1}{T_{BES\,4}}\Delta P_{BES_4} \\ \qquad +\frac{1}{T_{BES\,4}}(r_{BES_4}+\Delta r_{BES_4})\Delta f_4, \\ \Delta\dot{f}_4 = -\frac{2D_4}{M_4}\Delta f_4+\frac{2}{M_4}\Delta P_4. \end{cases} \tag{4}$$

Power generated by PVs and WTGs in each MG depends on the condition of light intensity and wind power, and the state of local load can change the dissipation of power energy. In this paper, v_{L_1}, v_{L_2}, v_{L_3},v_{L_4}, v_{PV_1}, v_{PV_2},v_{WTG_1} and v_{WTG_4} are denoted as the disturbance inputs for power changes of the local loads, PV unit and WTG, corresponding to their subscripts.

The randomness involved by loads, WTGs and PV units in each MG is described by standard Weiner process $W(t)$. We denote u_{FC_1},u_{MT_2},u_{FC_3},u_{MT_4}, $u_{ER_{12}}$,$u_{ER_{13}}$,$u_{ER_{23}}$ and $u_{ER_{43}}$ as the control inputs of their corresponding subscripts.

In the form of a combination of ODEs and SDEs, the dynamics of all elements in MG_1, MG_2, MG_3 and MG_4 areconsidered with state-delay and parameter uncertainties, formulated in (1), (2), (3), (4), respectively (time t omitted).

As shown in (5), we define $\Delta P_{ER_{12}}$, $\Delta P_{ER_{13}}$, $P_{ER_{23}}$, $\Delta P_{ER_{43}}$ as power change transmitted from MG_1 to MG_2,from MG_1 to MG_3 , from MG_2 to MG_3 and from MG_1 to MG_2 , respectively.Then we have

$$\begin{cases} \Delta\dot{P}_{ER_{12}} = \frac{1}{T_{ER\,12}}\Delta P_{ER_{12}}(t-\tau(t)) \\ \qquad +\frac{1}{T_{ER\,12}}(b_{ER_{12}}+\Delta b_{ER_{12}})u_{ER_{12}}, \\ \Delta\dot{P}_{ER_{13}} = -\frac{1}{T_{ER\,13}}\Delta P_{ER_{13}}(t-\tau(t)) \\ \qquad +\frac{1}{T_{ER\,13}}(b_{ER_{13}}+\Delta b_{ER_{13}})u_{ER_{13}}, \\ \Delta\dot{P}_{ER_{23}} = -\frac{1}{T_{ER\,23}}\Delta P_{ER_{23}}(t-\tau(t)) \\ \qquad +\frac{1}{T_{ER\,23}}(b_{ER_{23}}+\Delta b_{ER_{23}})u_{ER_{23}}, \\ \Delta\dot{P}_{ER_{43}} = -\frac{1}{T_{ER\,43}}\Delta P_{ER_{43}}(t-\tau(t)) \\ \qquad +\frac{1}{T_{ER\,43}}(b_{ER_{43}}+\Delta b_{ER_{43}})u_{ER_{43}}. \end{cases} \tag{5}$$

We denote the AC bus power of MG_1, MG_2, MG_3 and MG_4 as ΔP_1, ΔP_2, ΔP_3 and ΔP_4, respectively. The power balance equationswithin the whole considered EI are given in (6).

$$\begin{cases}\Delta P_1 = \Delta P_{PV_1} + \Delta P_{WTG_1} + \Delta P_{BES_1} + \Delta P_{FC_1} \\ \qquad -\Delta P_{L_1} - \Delta P_{ER_{12}} - \Delta P_{ER_{13}}, \\ \Delta P_2 = \Delta P_{PV_2} + \Delta P_{MT_2} + \Delta P_{BES_2} - \Delta P_{L_2} \\ \qquad +\Delta P_{ER_{12}} - \Delta P_{ER_{23}}, \\ \Delta P_3 = \Delta P_{FC_3} + \Delta P_{BES_3} - \Delta P_{L_3} + \Delta P_{ER_{13}} \\ \qquad +\Delta P_{ER_{23}} + \Delta P_{ER_{43}}, \\ \Delta P_4 = \Delta P_{WTG_4} + \Delta P_{MT_3} + \Delta P_{BES_3} - \Delta P_{L_3} - \Delta P_{ER_{43}}. \end{cases} \tag{6}$$

We introduce the time varying delay$\tau(t)$ in the equation containing the control signal, and we assume $0 < \tau(t) \le \mu < \infty$ and $\dot{\tau(t)} \le h < 1$where μ and h are real constant scalars. Such assumption has been used in many works; see, e.g., [21]–[23].

Time constants and the factors $r_{L_1}, r_{L_2}, r_{L_3}, r_{L_4}, r_{PV_1}, r_{PV_2}$, r_{WTG_1},r_{WTG_4}, r_{BES_1}, r_{BES_2}, r_{BES_3}, r_{BES_4},b_{MT_2},b_{MT_4},b_{FC_1},b_{FC_3}, $b_{ER_{12}}$,$b_{ER_{13}}$,$b_{ER_{23}}$, $b_{ER_{43}}$ in (1)-(5) depend on real engineering scenarios and can be measured by parameter estimation methods. It is notable thatthe measurement error is inevitable. For our considered EI system, parameter uncertainties are set in (1)-(5).Theparameter uncertainties of the factors r_{BES_1}, r_{BES_2}, r_{BES_3}, r_{BES_4},b_{MT_2},b_{MT_4}, b_{FC_1},b_{FC_3}, $b_{ER_{12}}$,$b_{ER_{13}}$,$b_{ER_{23}}$, $b_{ER_{43}}$ are defined as Δr_{BES_1}, Δr_{BES_2}, Δr_{BES_3}, Δr_{BES_4},Δb_{MT_2},Δb_{MT_4}, Δb_{FC_1},Δb_{FC_3}, $\Delta b_{ER_{12}}$,$\Delta b_{ER_{13}}$,$\Delta b_{ER_{23}}$, $\Delta b_{ER_{43}}$, repectively.We denote the otherparameter uncertainties for the system (particularly for time constant) as Δo_{L_1},Δo_{L_2},Δo_{L_3},Δo_{L_4},Δo_{PV_1},Δo_{PV_2},$\Delta o_{\mathrm{WTG}_1}$,$\Delta o_{\mathrm{WTG}_4}$,$\Delta o_{\mathrm{MT}_2}$, Δo_{MT_4},Δo_{FC_1},Δo_{FC_3},$\Delta o_{\mathrm{BES}_1}$,$\Delta o_{\mathrm{BES}_2}$,$\Delta o_{\mathrm{BES}_3}$,$\Delta o_{\mathrm{BES}_4}$,$\Delta o_{\mathrm{ER}_{12}}$, $\Delta o_{\mathrm{ER}_{13}}$,$\Delta o_{\mathrm{ER}_{23}}and\Delta o_{\mathrm{R}_{43}}$ in (1)-(5).For natation simplicity, let

$$\Delta O_1 = \begin{bmatrix}\Delta O_{L_1}\\ \Delta O_{PV_1}\\ \Delta O_{WTG_1}\\ \Delta r_{BES_1}\end{bmatrix}, \qquad \Delta O_2 = \begin{bmatrix}\Delta O_{L_2}\\ \Delta O_{PV_2}\\ \Delta r_{BES_2}\end{bmatrix},$$

$$\Delta O_3 = \begin{bmatrix}\Delta O_{L_3}\\ \Delta r_{BES_3}\end{bmatrix}, \qquad \Delta O_4 = \begin{bmatrix}\Delta O_{L_4}\\ \Delta O_{WTG_4}\\ \Delta r_{BES_4}\end{bmatrix}.$$

$$\Delta B_{MG} = \begin{bmatrix}\Delta b_{FC_1}\\ \Delta b_{MT_2}\\ \Delta b_{FC_3}\\ \Delta b_{MT_4}\end{bmatrix}, \qquad \Delta B_{ER} = \begin{bmatrix}\Delta b_{ER_{12}}\\ \Delta b_{ER_{13}}\\ \Delta b_{ER_{23}}\\ \Delta b_{ER_{43}}\end{bmatrix},$$

where $\Delta O = [\Delta O_1 \quad \Delta O_2 \quad \Delta O_3 \quad \Delta O_4]'$, and $\Delta B = [\Delta B_{MG} \quad \Delta B_{ER}]'$. The structure of the parameter uncertainty in this paper follows the form of

$$[\Delta O \Delta B] = MF(t)[N_o N_b], \tag{7}$$

where M, N_o and N_b are known real constant matricesdepending on engineering situation, and $F(\cdot)$ is an unknown time-varying matrix function satisfying

$$F(t)' F(t) \le I. \tag{8}$$

The form in (7)-(8) has been used in a variety of works; see, e.g., [21], [24], [25].

Inorderto simplify the equation set (1)-(7), we define some new notations as follows:

$$x_L = [\Delta P_{L_1} \quad \Delta P_{L_2} \quad \Delta P_{L_3} \quad \Delta P_{L_4}]',$$
$$x_{PV} = [\Delta P_{PV_1} \quad \Delta P_{PV_2}]',$$
$$x_{WTG} = [\Delta P_{WTG_1} \quad \Delta P_{WTG_4}]',$$
$$x_{MT} = [\Delta P_{MT_2} \quad \Delta P_{MT_4}]',$$
$$x_{FC} = [\Delta P_{FC_1} \quad \Delta P_{FC_3}]',$$
$$x_{BES} = [\Delta P_{BES_1} \quad \Delta P_{BES_2} \quad \Delta P_{BES_3} \quad \Delta P_{BES_4}]',$$
$$x_{ER} = [\Delta P_{ER_{12}} \quad \Delta P_{ER_{13}} \quad \Delta P_{ER_{23}} \quad \Delta P_{ER_{43}}]',$$
$$x_f = [\Delta f_1 \quad \Delta f_2 \quad \Delta f_3 \quad \Delta f_4]'.$$

Let

$$x = \begin{bmatrix}x_L\\ x_{PV}\\ x_{WTG}\\ x_{MT}\\ x_{FC}\\ x_{BES}\\ x_{ER}\\ x_f\end{bmatrix}, \qquad u = \begin{bmatrix}u_{FC_1}\\ u_{MT_2}\\ u_{FC_3}\\ u_{MT_4}\\ u_{ER_{12}}\\ u_{ER_{13}}\\ u_{ER_{23}}\\ u_{ER_{43}}\end{bmatrix}, \qquad v = \begin{bmatrix}v_{L_1}\\ v_{L_2}\\ v_{L_3}\\ v_{L_4}\\ v_{PV_1}\\ v_{PV_2}\\ v_{PV_4}\\ v_{WTG_1}\\ v_{WTG_4}\end{bmatrix},$$

and $z = x_f$. Observing (1)-(7), we have the following integrated stochastic system(time t omitted),

$$\begin{cases}\mathrm{d}x = [(A + \Delta O)x + A_d x\big(t - \tau(t)\big) + (B + \Delta B)u \\ \qquad + Cv]\mathrm{d}t + Rx\mathrm{d}W(t), \\ z = Dx. \end{cases} \tag{9}$$

Insystem (9), $x(t)$, $u(t)$and $v(t)$represent for the system state, control input and disturbance input, respectively. The controlled output of the whole system is the frequency deviation of each MG, which is denoted as $z(t)$ in (9). Here,Dsatisfies

$$D = \begin{bmatrix}O_{12\times 12} & O_{12\times 4}\\ O_{4\times 12} & I_4\end{bmatrix},$$

where I_n is an $n \times n$ identity matrix and$O_{m\times n}$ is an $m \times n$ zero matrix.

Taking into account (1)-(9), we have transformed the physical EI system into a mathematical control system.

III. Problem Formulation and Solution

In this section, we formulate the coordinated frequency control problem for EI as problems of robust stabilization and robust H_∞ control. Then we provide sufficient solutions to these problems.

First, we present the definition of robust stabilization.Let us denote $\mathbb{E}$as mathematical expectation.

Definition 1: (see, e.g., [21]) *The system (9) with* $u = 0$ *and* $v = 0$ *is said to be mean-square stable if for any* $\varepsilon > 0$ *there is a* $\delta(\varepsilon) > 0$ *such that*

$$\mathbb{E}|x(t)|^2 < \varepsilon, \qquad t > 0,$$

when

$$\sup_{-\mu \le s \le 0} \mathbb{E}|\emptyset(s)|^2 < \delta(\varepsilon), \qquad t > 0.$$

If, in addition

$$\lim_{t\to\infty} \mathbb{E}|x(t)|^2 = 0,$$

for any initial conditions, then system (9) with $u = 0$ and $v = 0$ is said to be mean-square asymptotically stable. The uncertain stochastic system in (9) is said to be robustlystochastically stable if the system associated to (9) with $u = 0$ and $v = 0$ is mean-square asymptotically stable for all the system parameter uncertainties.

Next, the definition of robust H_∞ performance for our considered coordinated frequency control problem is presented as follows.

Definition 2: (see, e.g., [21])*Given a scalar $\gamma > 0$, theH_∞ performance for the coordinated frequency regulation problem is defined as $||z(t)|| < \gamma||v(t)||$. Here, norm $\|\cdot\|$ is defined as*

$$\|z(t)\| \triangleq \left(\mathbb{E}\left\{\int_0^\infty |z(t)|^2 \mathrm{d}t\right\}\right)^{1/2},$$

where scalar γ is called disturbance attenuation. Based on theH_∞ performance introduced above, the stochastic H_∞ cost functional is formulatedas follows,

$$J(u,v) \triangleq \mathbb{E}\left[\int_0^T (z'z - \gamma^2 v'v)\mathrm{d}t\right].$$

Before we present the main results, the following lemma is necessary.

Lemma 1:(see, e.g., [26])*Let D, S and F be real matrices of appropriate dimensions such that $F^T F \le I$. Then, for scalar $\epsilon > 0$ and vectors $x, y \in \mathbb{R}^n$*

$$2x^T DFSy \le \epsilon^{-1} x^T DD^T x + \epsilon y^T S^T Sy.$$

First, the stochastic stabilization problem is solved in Theorem 1.

Theorem 1:(see, e.g., [21])*Consider the uncertain stochastic delay system (9) with $v = 0$. This system is robustly stochastically stabilizable ifthere exist scalars $\epsilon_1 > 0$ and matrices $X > 0, S > 0$ and Y such that the following LMI holds,*

$$\begin{bmatrix} \Gamma & A_d X & XN_o{}' + Y'N_b{}' & XR' \\ XA_d{}' & (h-1)S & 0 & 0 \\ N_o X + N_b Y & 0 & -X & 0 \\ RX & 0 & 0 & -\varepsilon_1 I \end{bmatrix} \le 0,$$

where

$$\Gamma = AX + XA' + BY + Y'B' + S + \varepsilon_1 MM'.$$

In this case, an appropriate state feedback controller can be chosen by $u^* = Kx, K = YX^{-1}$.

Based on Theorem 1, the stochastic robust H_∞ control stratrgy is to find a controller u^* for system (9), such that for all nonzero disturbance $v(t)$,$J(u^*, v) \le 0$ holds.

Theorem 2:(see, e.g., [21]) *Consider the uncertain stochastic delay system (9).Given a scalar $\gamma > 0$, then this system is robustly stochastically stabilizablewith disturbance attenuation γ if there exist scalars $\epsilon_1 > 0$ and matrices $X > 0, S > 0$ and Y such that the following LMI holds*

$$\begin{bmatrix} \Gamma & XD' & XR' & C & XN_o{}' + Y'N_b{}' & A_d X \\ DX & -I & 0 & 0 & 0 & 0 \\ RX & 0 & -X & 0 & 0 & 0 \\ C' & 0 & 0 & -\gamma^2 I & 0 & 0 \\ N_o X + N_b Y & 0 & 0 & 0 & -\varepsilon_1 I & 0 \\ XA_d{}' & 0 & 0 & 0 & 0 & (h-1)S \end{bmatrix} \le 0.$$

Inthis case, an appropriate state feedback controller can be chosen by $u^* = Kx, K = YX^{-1}$.

IV. STOCHASTICH_∞ CONTROL PROBLEM FORMULATION AND SOLUTION

In this section, several typical simulation results are presented to demonstrate the feasibility of the proposed robust H_∞control scheme. Based on real world data, the parameters of the considered EI system (1)-(9) are shown in Table I and Table II. For the stochastic robustH_∞ control problem, we assume$\gamma = 0.3$, $\varepsilon_1 = 0.5$, $h = 0.8$ and $\mu = 0.8$. The matrices M, N_o, and N_b describing the system uncertainty of system (9) are randomly generated from three normal distributions. The LMI problem in Theorem 1 and Theorem 2 are solved with MATLAB LMI ControlToolbox.

TABLE I

TIME CONSTANTS OF THE CONSIDERED EI SYSTEM

Parameter	Value (s)	Parameter	Value (s)
T_{L_1}	1.3	T_{L_2}	2.1
T_{L_3}	1.8	T_{L_4}	1.6
T_{PV_1}	1.4	T_{PV_2}	0.6
T_{WTG_1}	2.1	T_{WTG_4}	1.6
T_{BES_1}	0.14	T_{BES_2}	0.16
T_{BES_3}	0.12	T_{BES_4}	0.11
T_{FC_1}	1.1	T_{FC_3}	1.2
T_{MT_2}	1.3	T_{MT_4}	1.5
$T_{ER_{12}}$	0.1	$T_{ER_{13}}$	0.11
$T_{ER_{23}}$	0.12	$T_{ER_{43}}$	0.16

TABLE II

PARAMETERS OF THE CONSIDERED EI SYSTEM

Parameter	Value	Parameter	Value
r_{L_1}	0.9	r_{L_2}	0.8
r_{L_3}	0.7	r_{L_4}	0.6
r_{PV_1}	0.8	r_{PV_2}	0.9
r_{WTG_1}	0.6	r_{WTG_4}	0.9
r_{BES_1}	1.2	r_{BES_2}	1.1
r_{BES_3}	1.4	r_{BES_4}	1.2
b_{FC_1}	2.6	b_{FC_3}	1.5
b_{MT_2}	3.1	b_{MT_4}	2.8
$b_{ER_{12}}$	1.1	$b_{ER_{13}}$	1.2
$b_{ER_{23}}$	1.4	$b_{ER_{43}}$	1.3

D_1	0.012	M_1	0.20
D_2	0.010	M_2	0.19
D_3	0.020	M_3	0.22
D_4	0.016	M_4	0.21

Firstly, we show that system (9) is effectively stabilized with the proposed stochastic robust stabilizing controller in Theorem 1. The disturbance inputs of system (7) is set to be zero. The initial value of state variable x is set to be a small non-zero vector. As shown in Fig. 2, system (7) with uncertain parameters tends to be divergent rapidly when no control signal is implemented.

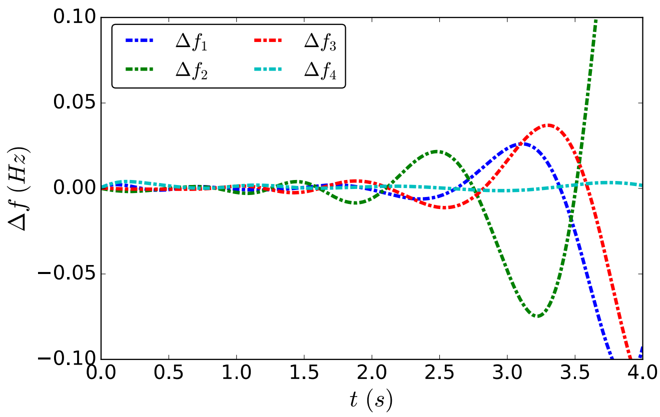

Fig.2. Frequency deviations without control (no disturbance input).

In Fig. 3, we see that the frequency deviations in four MGs are successful stabilized when controller obtained via Theorem 1 is applied into system (9).

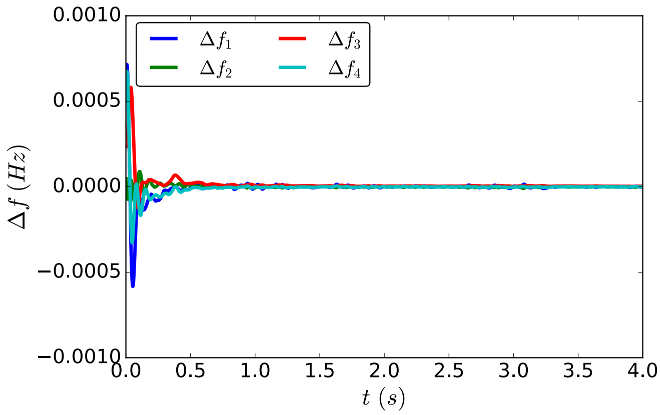

Fig.3. Frequency deviations under robust controller (no disturbance input).

Next, let us evaluate the effectiveness of the coordinated frequencycontrollerobtained in Theorem 2. The disturbance inputs for system (7) are presented in Fig. 4. The corresponding power fluctuations of the uncontrollable components including loads, PV units, and WTGs are illustrated in Fig. 5 and Fig. 6, respectively.

When no control scheme is applied, the oscillations of the frequency deviations on AC buses in the four MGs are shown in Fig. 7. Whereas the frequency oscillations of the four MGs under the proposed stochastic robustH_∞ control strategy are presented in Fig. 8. Obviously, the controller designed in Theorem 2 achieves the frequency stabilization effectively.

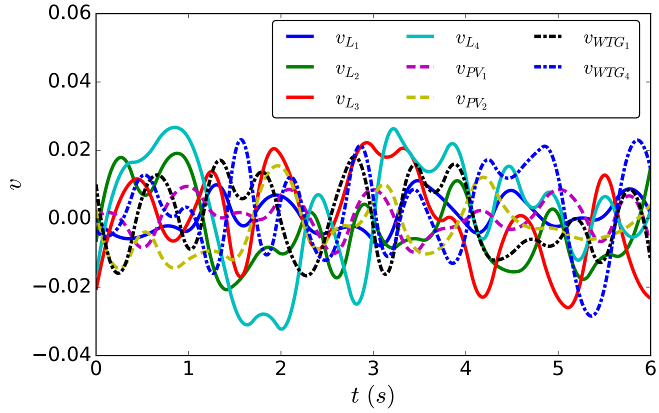

Fig.4. Disturbance inputs for the EI system.

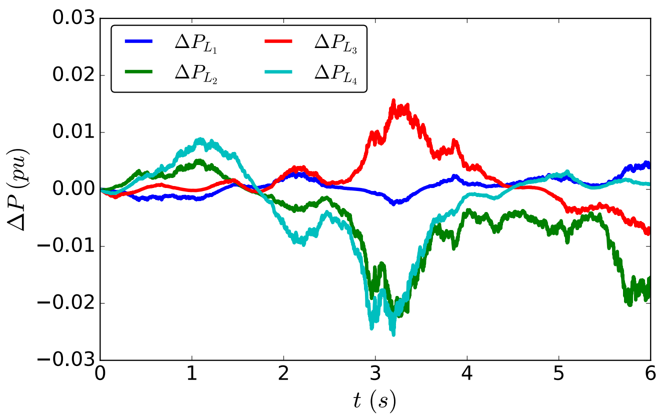

Fig. 5. Power deviations of loads.

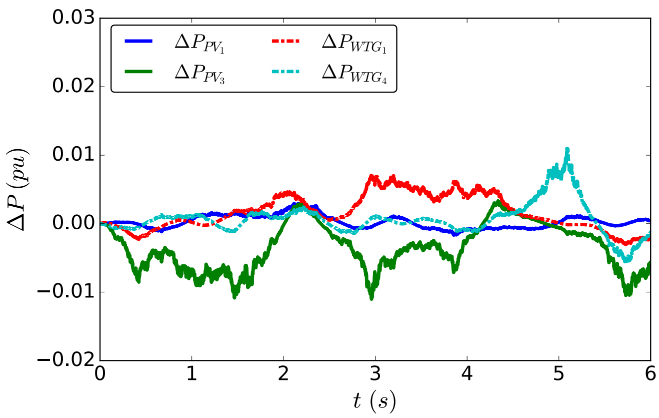

Fig. 6. Power deviations of PV units and WTGs.

Fig. 9 illustrates the power generation change of FCs and MTs under the proposed H_∞ controller. The fluctuations of the energy transmitted over ERs between MGs are presented in Fig. 10. From Fig. 5, Fig. 6, Fig.9 and Fig.10, we see that the power usage of loads and power generation from PV units and WTGs in EI could be balanced efficiently with the help of ERs.

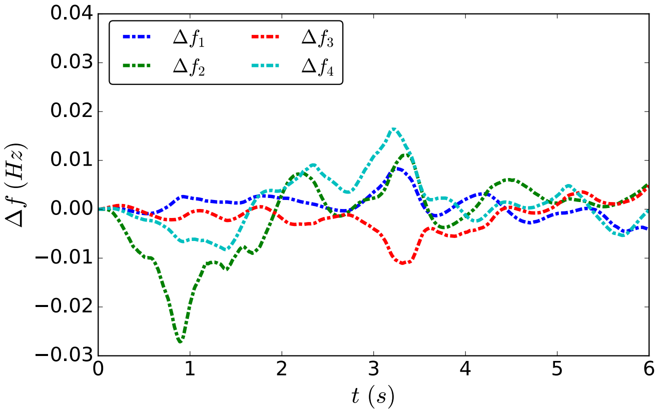

Fig.7. Frequency oscillations without control (with disturbance input).

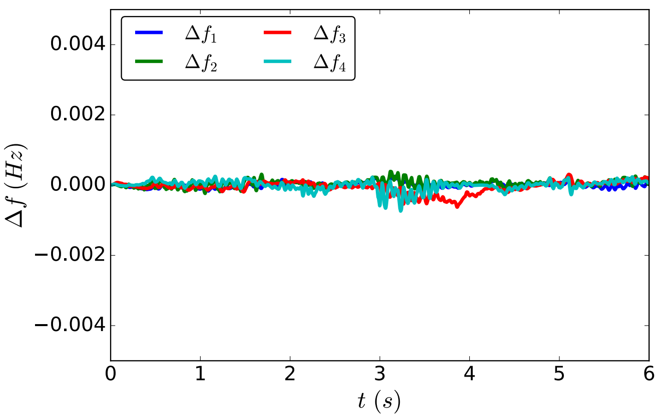

Fig.8. Frequency oscillations with robust H_∞ controller (with disturbance input).

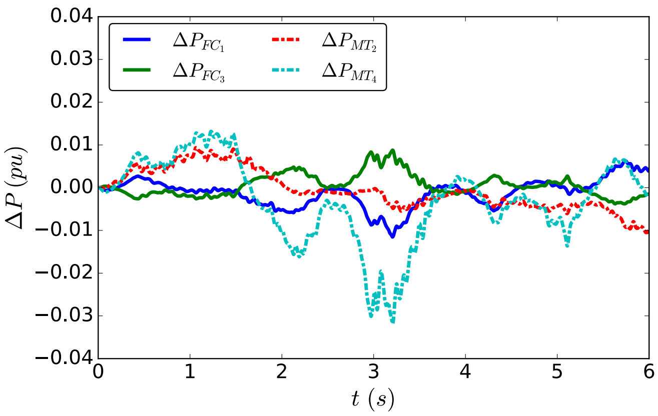

Fig. 9. Power deviations of FCs and MTs.

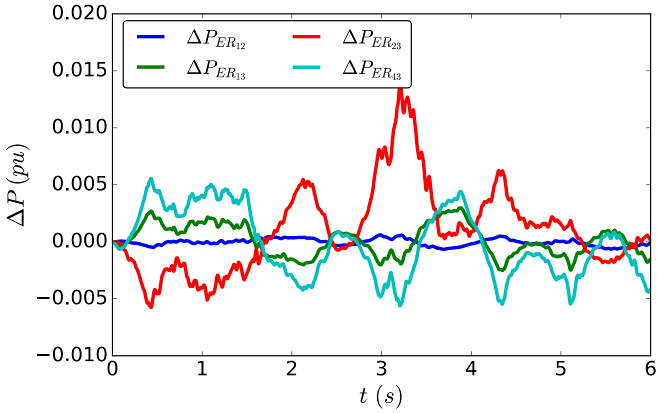

Fig. 10. Power deviations of ERs.

By evaluating the controllers obtained from Theorem 1 and Theorem 2 with numerical simulations, the effectiveness of the our proposed method is demonstrated distinctly.

V. Conclusion

In summary, an application of modern robust H_∞ control theory for uncertain stochastic systems with state delay into the scenario of EI is investigated in this paper. The complicated engineering scenario of EI is modeled into a class of mathematical system and solved by related stochastic control theory. Focusing on the popular topic of EI, we emphasizethe contribution of this work is mainly on the system modeling and problem formulation. Throughout the control problems in power systems, this is the very first time that system stochasticity, time delay and parameter uncertainty is considered simultaneously for each component in EI. In the future, in addition to the coordinated frequency control problem, we will also focus on the problem of optimal operation cost management for the scenario of EI.

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