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MULTIPLE SIGNAL CLASSIFICATION FOR GRAVITATIONAL WAVE BURST SEARCH

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This work is mainly focused on the application of the multiple signal classification (MUSIC) algorithm for gravitational wave burst search. With an array of gravitational wave detectors with arbitrary location and directional characters in a certain environment with arbitrary noise covariance, major characteristics of gravitational waves can be obtained using the MUSIC algorithm.

In this work, the principle of the MUSIC algorithm is described in details. Extensions are made for gravitational wave burst serach. Performance evaluation is carried out using self-created and simulated signals and noises. MUSIC is compared with Q-transform for signal triggering and Bayesian for direction of arrivals (DOA) estimation using the Ω-pipeline. Experimental results show that MUSIC can be processed faster, though with lower resolution. MUSIC has potential to be applied for real-time gravitational wave search for multi-messenger astronomy.

Keywords: MUSIC; gravitational wave burst; direction of arrivals; multi-messenger astronomy.

1. Introduction

The era of gravitational wave (GW) astronomy is coming with several detectors reaching their sensitivity in several years, such as Advanced LIGO[1,2] (Laser Interferometer Gravitational-wave Observatory), the Virgo[3] GW detector, and LCGT[4] (Large-scale Cryogenic Gravitational wave Telescope). The network of GW detectors is essential for gravitational wave search.

Multiple signal classification[5] (MUSIC) is a well-established method to process signals obtained by a series of sensors first brought up by Ralph O. Schmidt in

1986. Giving arbitrary locations and directional characteristics in a noisy environment with arbitrary covariance, MUSIC is capable of giving asymptotically unbiased estimation of:

- Number of signals;
- Direction of arrivals (DOA)
- Strengths and cross correlations among the directional waveforms;
- Polarizations;
- Strength of noise or interference.

The MUSIC method yields a rather high SNR ratio and has no restrict on wave form theoretically. The GW network could be treated as various sensors so that MUSIC could be applied. If a gravitational wave burst occurs and passed through earth, it is meant to be recorded by various detectors simutaneously. A numerical method is required for joint GW data analysis. With the development of GW detection technology, the SNR ratio is dramatically improved in the past decade, however, still not well enough, so calling for a sensitive enough data analysis approach. Although several types of GW forms can be modeled, most others are not predictable. The numerical method cannot have much pre-determined knowledge of GW forms. As a result, MUSIC draws our attention for its satisfaction of all these conditions.

The rest of the paper is organized as follows: The principle of the MUSIC algorithm and its extensions for GW search are described in details in Section 2; In Section 3, Performance evaluation is carried out using self-created GW signals and noises in Section 3; In Section 4, MUSIC is compared with Q-transform[6] for signal triggering and Bayesian[7] for DOA estimation using the Ω-pipeline[8]. Experimental results show that MUSIC can be processed faster, though with lower resolution. MUSIC has potential to be applied for real-time GW search for multi-messenger astronomy[9], which is concluded in Section 5.

2. The MUSIC method

2.1. *The MUSIC algorithm*

The following shows the data structure of MUSIC:

$$\begin{aligned} X &= AF + W \\ X &= [X_1\ X_2\ ...\ X_M]^T \\ A &= [a(\theta_1)\ a(\theta_2)\ ...\ a(\theta_D)] \\ W &= [W_1\ W_2\ ...\ W_M]^T \end{aligned} \tag{1}$$

Where M represents the number of detectors, D represents the number of signals, namely $M > D$. X is the signal vector and A is the mode vector concerning about antenna response. F represents the source signal and W describes noises.

Considering the covariance matrix of X we get:

$$\begin{aligned} S = \overline{XX^*} = \overline{(AF+W)(AF+W)^*} &= \overline{(AF+W)(F^*A^*+W^*)} \\ = A\overline{FF^*}A^* + A\overline{FW^*} + \overline{WF^*}A + \overline{WW^*} &= A\overline{FF^*}A^* + \overline{WW^*} \end{aligned} \tag{2}$$

Or

$$S = APA^* + \lambda S_0 \tag{3}$$

Where P is a $D\times D$ positive definite diagonal matrix, A is a $M\times D$ full rank matrix, so APA^* is nonnegative definite, the rank of which is D. So the number of non-zero eigenvalues is D and the number of zero eigenvalues is M-D. As a result:

$$|APA^*| = |S - \lambda S_0| = 0 \tag{4}$$

As shown above, λ is an eigenvalue of S in the matric of S_0. In order to insure APA^* is nonnegative definite, λ must be the smallest eigenvalue of S.

So we can get:

$$S = APA^* + \lambda_{min} S_0 \tag{5}$$

In the case of white Gaussian noises:

$$\lambda_{min} S_0 = \sigma^2 I \tag{6}$$

For the number of zero eigenvalues of APA^* is M-D, so S must have minimum eigenvalues repeated M-D times, say the number is N. So:

$$\widetilde{D} = M - \widetilde{N} \tag{7}$$

That is the estimation of signal number. For λ_{min} is the N times repeated minimum eigenvalues of S in the metric of S_0, so E is N linearly independent eigenvectors e_i, each one satisfies:

$$Se_i = \lambda_{min} S_0 e_i \tag{8}$$

S is multiplied by e_i:

$$Se_i = APA^* e_i + \lambda_{min} S_0 e_i \tag{9}$$

As a result:

$$APA^* e_i = 0 \tag{10}$$

Or we can get:

$$A^* e_i = 0 \tag{11}$$

If we using all the vectors from A to form a RD signal subspace and using N linearly independent eigenvectors e_i to form a noise subspace, the two subspaces are perpendicular to each other. Considering the Euclidean distance between them:

$$d^2 = Y^* E_N E_N^* Y \tag{12}$$

We can get:

$$P_\mu(\theta) = \frac{1}{a^*(\theta) E_N E_N^* a(\theta)} \tag{13}$$

When θ comes close to DOA, P_μ proceeds positive infinite.

2.2. *MUSIC extensions*

The MUSIC method is widely used in periodic sine radio waves detection by antenna arrays in the plane condition. Several aspects are extended before applying the MUSIC method for detection of gravitational waves.

- Using spherical coordinates to extend from a 2D into 3D space;
- Using the concept of equal-phase to extend linearly arrayed to generally placed detectors;
- Using linear transfer in time domain to extend the method to non-periodic signals.

The following are the steps of extended MUSIC algorithm for GW detection:

(i) Collecting data to form the covariance matrix S;
(ii) Calculating the Eigen structure of S in the metric of S_0;
(iii) Assuming that there is one signal in a relatively long period of time, getting the eigenvectors of noise subspace with the number of M-1;
(iv) Calculating $P_\mu(\theta)$ and put it in a figure;
(v) Finding out the peak of the signal;
(vi) Getting DOA and other information of interest.

3. Performance evaluation

As shown in Fig. 1, self-made signal containing white Gaussian noise and Gaussian pulse are used to evaluate MUSIC performance.

The results under different SNR ratios are shown in Fig. 2. The real signal position is fi=1.05 and si=1.05. Fig. 2a is the MUSIC spectrum where SNR is 7.5 dB and the calculated signal position is fi=1.08 and si=1.1; Fig. 2b is the MUSIC spectrum where SNR is 1.5 dB and the calculated signal position is fi=1.1 and

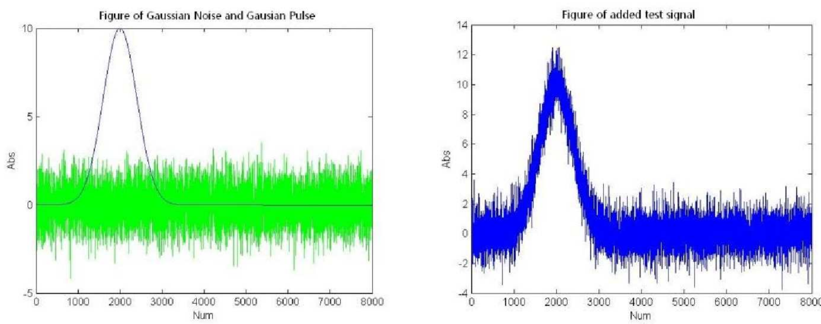

Fig. 1. Test signal added from white noise and Gaussian pulse.

si=1.02; Fig. 2c is the MUSIC spectrum where SNR is -4 dB and the calculated singal position is fi=1.02 and si=0.7. In these scenarios, MUSIC performance is pretty good. Fig. 2d is the MUSIC spectrum where SNR is -8.8 dB and the calculated signal position is fi=0.5 and si=0.25, which is pretty bad. So the SNR ratio limitation of MUSIC is around -5 dB.

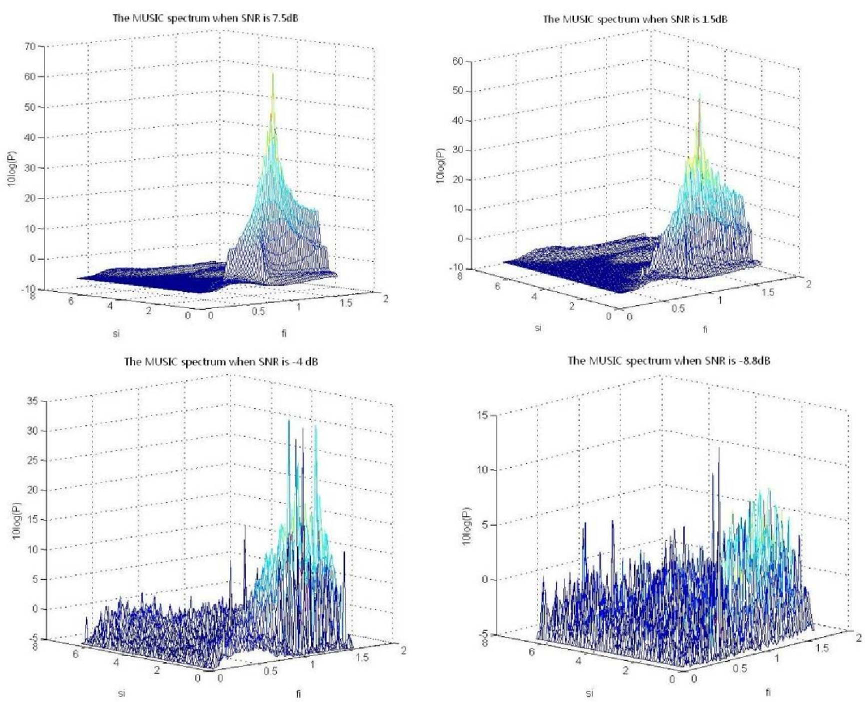

Fig. 2. The MUSIC spectrum of various SNR ratios, from up right to down left are a b c and d.

4. Performance comparison

MUSIC is compared with Ω-pipeline in this work, with self-made gravitational signal injected into simulated background noises. Two experiments are carried out for signal triggering and DOA estimation.

At First, define A as the relative signal strength, which comes from the parameter of Factor of LogFile of the injection part. A typical gravitational wave has a strength A=1.

Experimental results of MUSIC for signal triggering are shown in Table 1, compared with Q-transform. MUSIC performs bad for signal triggering.

Table 1. Performance comparison results for signal triggering.

Method	Low Limit of A	Time Consuming	Time resolution
Q-transform	2	14s	0.015s
MUSIC	200	3200s	0.03s

Experimental results of MUSIC for DOA estimation are shown in Table 2, compared with Bayesian. MUSIC is processed faster for DOA estimation, though with lower resolution.

Table 2. Performance comparison results for DOA estimation.

Method	Low Limit of A	Time Consuming	Angel resolution
Bayesian	4	30s	0.019rad
MUSIC	1000	4.2s	Complicated

The skymap of Bayesian when A=100 is illustrated in Fig. 3.

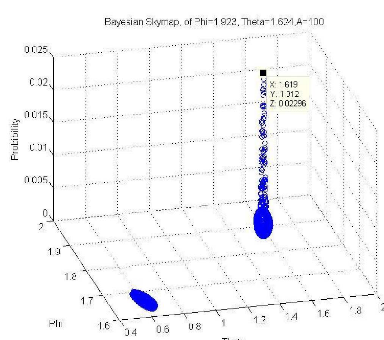

Fig. 3. Bayesian skymap when A=100.

And skymaps of MUSIC when A=1000 and A=5000 are shown in Fig. 4.

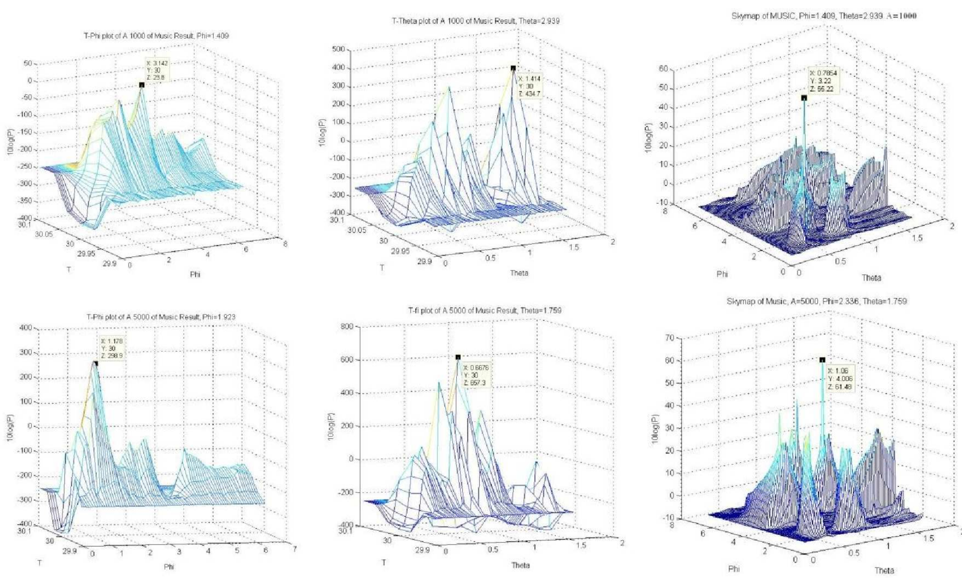

Fig. 4. T-ϕ (left), T-θ (middle) and sky-map (right) plots of MUSIC.

Skymaps show that sensitivity, stability and quality of MUSIC is far less than those of Bayesian even when the signal strength for MUSIC is tens of times larger.

5. Conclusions

According to our research, the MUSIC algorithm is a potential method for gravitational wave burst search, for it has no restrict on the signal form and can trigger out detection given DOA and other useful information in a fast speed.

Compared with existing tools, e.g Ω-pipeline, although MUSIC has the advantage of high speed, the low limit SNR ratio resolution and time resolution and stability make MUSIC a better choice only in special scenarios when processing speed for DOA estimation becomes a critical issue, e.g. for real-time GW search for multi-messenger astronomy[9].

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Appendix A. Performance data of Bayesian

Table 3. The performance data of Bayesian.

N	A	ϕ_0	θ_0	ϕ_b	θ_b	$\Delta\phi$	$\Delta\theta$	T_C
1	2	0.236	1.667	0.754	2.813	0.518	1.146	-
2	2	3.110	1.993	3.130	2.070	0.020	0.077	-
3	4	0.871	2.380	0.770	2.192	0.101	0.188	-
4	4	2.885	1.647	2.884	1.616	0.001	0.031	-
5	4	4.88	3.00	4.95	2.91	0.07	0.09	-
6	100	1.143	1.989	1.110	1.952	0.033	0.037	23.3s
7	100	0.637	2.655	0.627	2.647	0.010	0.008	49.8s
8	100	1.652	1.706	1.650	1.727	0.002	0.021	16.0s

Appendix B. Performance data of MUSIC

Table 4. The performance data of MUSIC.

N	A	ϕ_0	θ_0	ϕ_b	θ_b	$\Delta\phi$	$\Delta\theta$
1	200	0.351	1.811	0	1.355	-	-
2	200	0.054	1.518	0	1.316	-	-
3	500	1.698	1.425	0.864	0.490	-	-
4	500	4.800	1.012	0.864	0.668	-	-
5	1000	2.572	1.292	2.435	1.355	-	0.063
6	1000	4.927	1.145	1.963	1.414	-	0.269
7	1000	3.764	1.387	2.513	1.237	-	0.150
8	5000	4.184	1.104	0.314	1.119	-	0.015
9	5000	5.588	1.160	2.435	1.355	-	0.195
10	5000	1.377	0.512	6.283	0.511	-	0.001

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